Enrollment No: _____ Exam Seat No: _____

C.U.SHAH UNIVERSITY

WinterExamination-2015

Subject Name:Numerical Analysis

Subject Code: 4SC03MTE1 **Branch: B.Sc. (Mathematics,Physics)**

Semester: 3 Date: 08/12/2015 Time: 02:30 To 05:30 Marks: 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) Subtract the approximate numbers 57.4662 from 786.85.

b) Round off the following numbers correct to four significant figures:

$i)\ 5.2056, \quad ii)\ 0.0055672$

c) Construct a difference table for $y = x^3 + 2x + 1$, for $x = 1, 2, 3, 4, 5$.

d) Prove that $\Delta \cdot \nabla = \Delta - \nabla$.

e) Write Lagrange's inverse interpolation formula.

f) Construct a divided difference table forthe following data

x	5	15	22
y	7	36	160

g) If $y = 4x^6 - 5x$, find the percentage error in y at $x = 1$, if the error in $x = 0.04$.

h) Prove that $\mu = \frac{1}{2}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right]$.

i) Write Stirling's interpolation formula.

j) Bessel's formula is most appropriate when p lies between…… .

k) Gauss forward interpolation formula involves odd difference below the central line and even differences on the central line. Determine whether the statement is True or False?

l) The n^{th} divided difference of a polynomial of degree n is zero. Determine whether the statement is True or False?

m) Divided differences are not symmetric functions of their argument. Determine whether the statement is True or False?

n) If the interval of differencing be unity, then $\Delta^n[x]^n = n!$. Determine whether the statement is True or False?

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) If $u = \frac{5x^3z^3}{3y^2}$ and error x, y, z are $0.001, 0.0001, 0.000001$. Find relative maximum error in u at $x = 1, y = 2$ and $z = 3$. **(05)**

b) Determine $y(12)$ by Lagrange interpolation from the following values. **(05)**

x	11	13	14	18	20	23
y	25	47	68	82	102	124

c) Write down the approximate representation of $\frac{2}{3}$correct to four significant figures and then find: i) Absolute error, ii) Relative error, iii) Percetage error. **(04)**

Q-3 **Attempt all questions** **(14)**

a) Given the following table: **(05)**

x	0	5	10	15	20
$f(x)$	1.0	1.6	3.8	8.2	15.4

Construct the difference table and compute $f(21)$ by using Newton's Backward formula.

b) Using Newton's divided difference formula, find $f(x)$ from the following data **(05)**

x	0	2	3	4	6	7
$f(x)$	0	8	0	-72	0	1008

c) Show that **(04)**

$i)\ \Delta[f(x) \cdot \phi(x)] = f(x+h) \cdot \Delta\phi(x) + \phi(x) \cdot \Delta f(x)$,

$$ii)\ \Delta\left[\frac{f(x)}{\phi(x)}\right] = \frac{\phi(x) \cdot \Delta f(x) - f(x) \cdot \Delta \phi(x)}{\phi(x+h)\phi(x)}, [\phi(x) \neq 0]$$

Q-4 **Attempt all questions** **(14)**

a) Show that : **(05)**

$i)\ \Delta^n u_{x-n} = u_x - n\,u_{x-1} + \frac{n(n-1)}{2}u_{x-2} - \cdots + (-1)^n u_{x-n},$

$ii)\ e^x\left(u_0 + x\Delta u_0 + \frac{x^2}{2!}\Delta^2 u_0 + \cdots\right) = u_0 + xu_1 + \frac{x^2}{2!}u_2 + \cdots$

b) From the following table, estimate the number of students who obtain mark 40 to 45 **(05)**

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

c) Find $f(x)$, when its first difference is$x^3 + 4x^2 + 2x + 7$. **(04)**

Q-5 **Attempt all questions** **(14)**

a) State and prove Newton's forward interpolation formula. **(07)**

b) From the following table, find the value of $e^{1.17}$ using Gauss's backward interpolation formula. **(05)**

x	1.00	1.05	1.10	1.15	1.20	1.25	1.30
y	2.7183	2.8577	3.0042	3.1582	3.3201	3.4903	3.6693

c) Show that $\Delta^n[ke^{ax}] = k\,(e^{ah} - 1)^n e^{ax}$. **(02)**

Q-6 **Attempt all questions** **(14)**

a) State and prove Bessel's interpolation formula. **(07)**

b) From the following table, find the value of x when $y = 13.5$, by using inverse interpolation formula **(05)**

x	93	96.2	100.0	104.2	108.7
y	11.38	12.80	14.70	17.07	19.91

c) Prove that$\Delta \log f(x) = \log\left[1 + \frac{\Delta f(x)}{f(x)}\right]$. **(02)**

Q-7 **Attempt all questions** **(14)**

a) State and prove Newton's Divided difference interpolation formula. **(05)**

b) Use Lagrange's formula to express the function $\frac{x^2+x-3}{x^3-2x^2-x+2}$ as a sum of partial fractions. **(05)**

c) Evaluate the missing terms in the following table. **(04)**

x	0	1	2	3	4	5
$f(x)$	0	?	8	15	?	35

Q-8 **Attempt all questions** **(14)**

a) State and prove Lagrange's interpolation formula. **(05)**

b) From the following table, find $f(34)$ using Laplace Everett's formula. **(05)**

x	20	25	30	35	40
$y = f(x)$	11. 4699	12.7834	13.7648	14.4982	15.0463

c) If $f(x) = \frac{1}{x^2}$, find the divided differences $[a, b]$ and $[a, b, c]$. **(04)**